LECTURE 1:
Basics on Cosmic Ray Propagation

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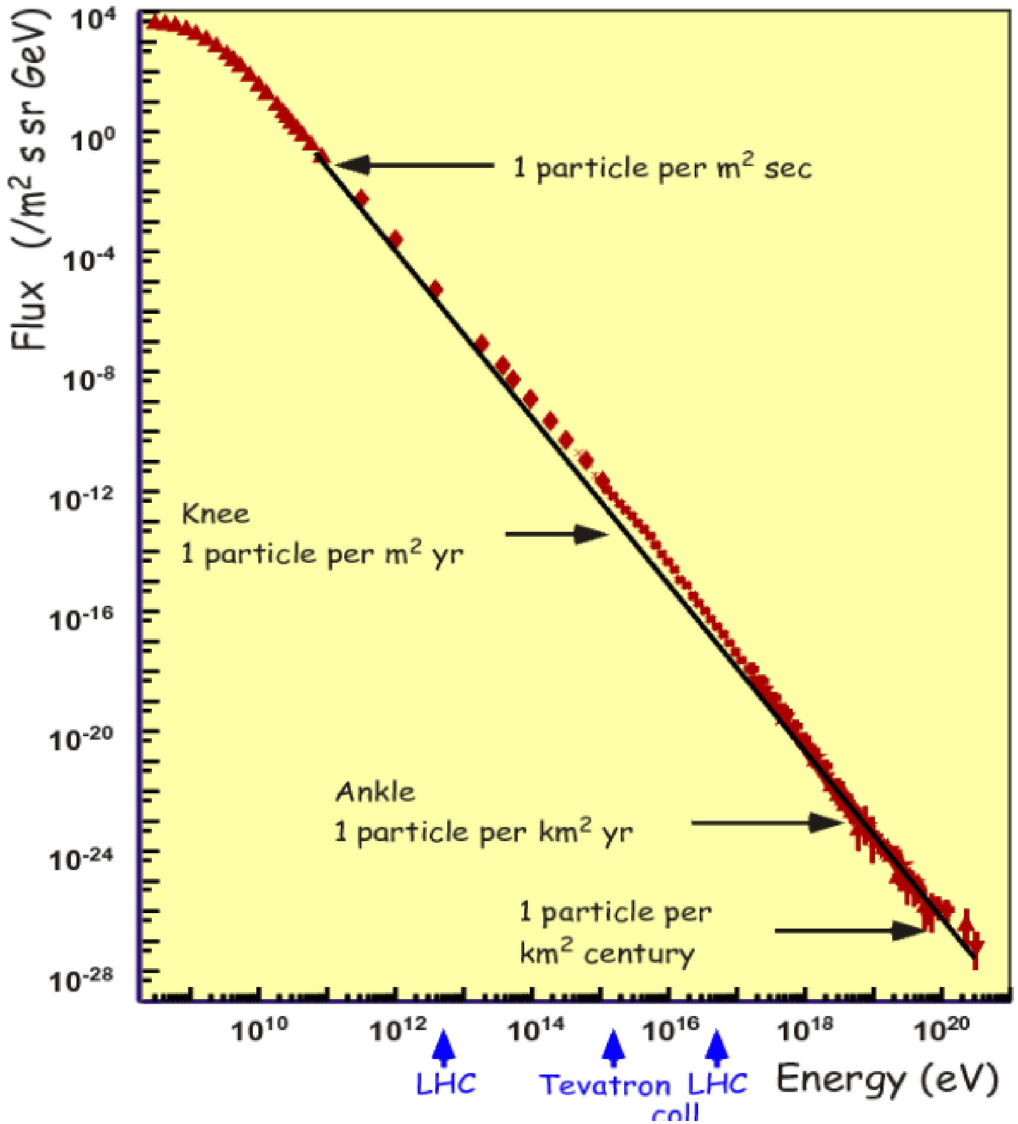
1 a –
Some observed properties of cosmic rays

Cosmic rays arriving unperturbed at the Earth's atmosphere are usually called ***primary cosmic rays***:

- **electrons** $\sim$ 1%
- **protons** $\sim$ 89%
- **heavier nuclei**, mainly helium $\sim$ 10%
- very few: antiparticles, muons, pions, kaons (from interactions of CRs with the interstellar gas)

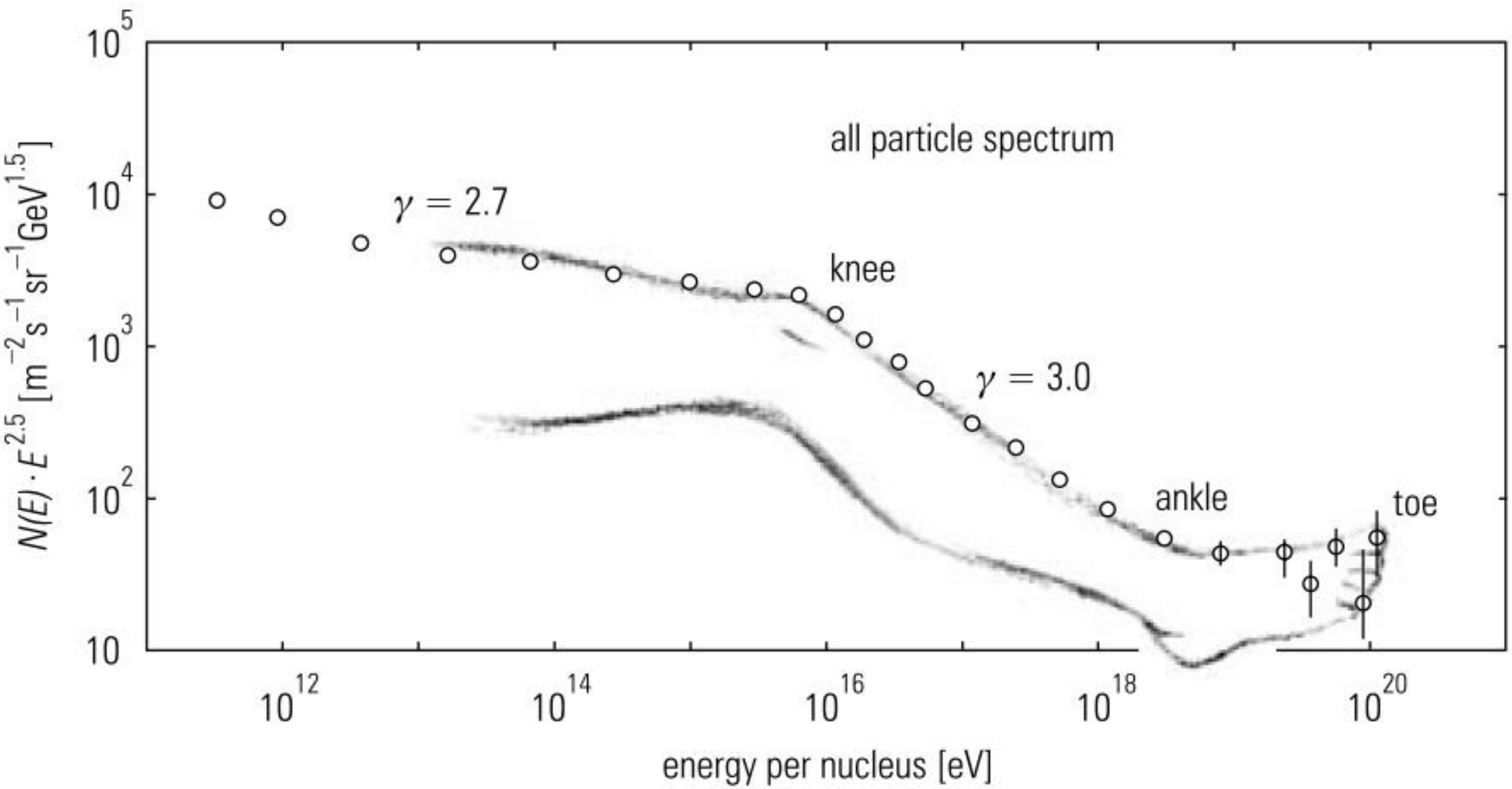
CR SPECTRUM - 1



- **power law**:
  $N(E) \propto E^{-\gamma}$

CR SPECTRUM - 2

The all-particle spectrum of charged primary cosmic rays is relatively steep so that practically no details are observable. Only after multiplication of the intensity with a power of the primary energy, structures in the primary spectrum become visible.

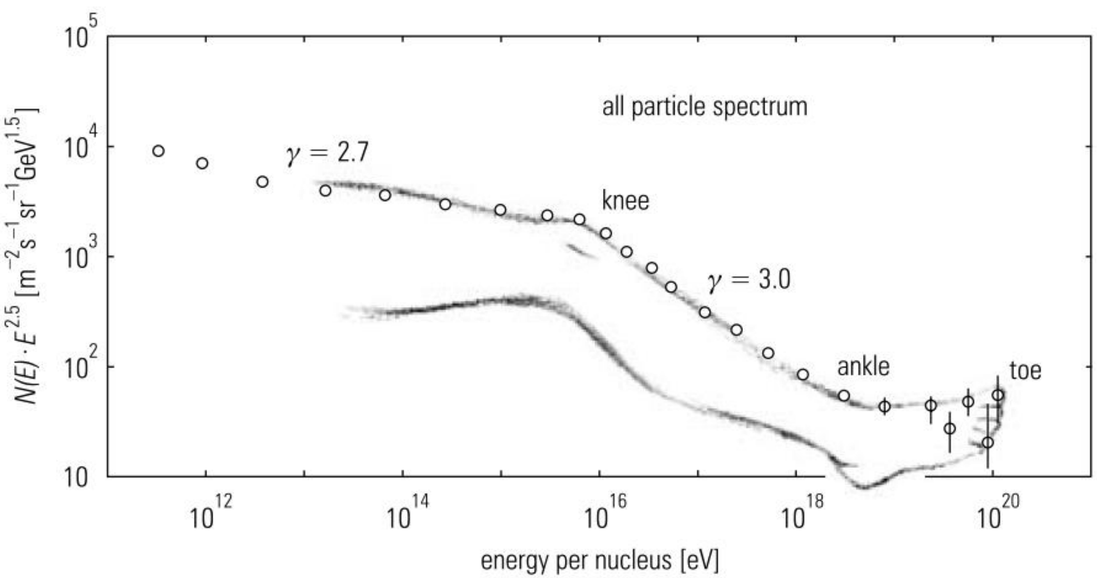


CR SPECTRUM - 3

• **KNEE:** cosmic rays with E > 10^{15} eV start to leak from the galaxy. Maximum from supernova explosions near 10^{15} eV

• **ANKLE:** Above the so-called ‘ankle’ at energies around 5 × 10^{18} eV the spectrum flattens again. This latter feature is often interpreted as a crossover from a steeper galactic component to a harder component of extragalactic origin.

• **GZK cutoff** at 6 x 10^{19} eV : cosmic rays above the energy of approximately 6 × 10^{19} eV would interact with the cosmic blackbody radiation: $\gamma + p \rightarrow p + \pi^0$, $\gamma + p \rightarrow n + \pi^+$. This limits the mean free path of high-energy protons to something like 10 Mpc.



1 b –
Particle motion in a constant magnetic field.

Cosmic rays are produced (and accelerated) at some astrophysical objects and before arriving to the Earth they propagate through the ***B*** *of the interstellar medium (**B** ~ 10^{-6} G) or the intergalactic medium (**B** ~ 10^{-9} G).*

Motion of a charged particle in a constant magnetic field

For $\vec{E} = 0$ and $\vec{B}$ = constant the equation of motion is:

$$m \frac{d\vec{v}}{dt} = q\,(\vec{v} \times \vec{B})$$

Taking $\vec{B} = B\,\hat{z}$ we have

$$m\,\dot{v}_x = q\,v_y\,B \qquad m\,\dot{v}_y = -q\,v_x\,B \qquad m\,\dot{v}_z = 0$$

These equations can be written as

$$\ddot{v}_x = \frac{qB}{m}\,\dot{v}_y = -\left(\frac{qB}{m}\right)^2 v_x$$

$$\ddot{v}_y = -\frac{q \cdot B}{m}\,\dot{v}_x = -\left(\frac{qB}{m}\right)^2 v_y$$

This describes a simple harmonic oscillator at the cyclotron frequency

$$\omega_c \equiv \frac{|q|\,B}{m}$$

The solution of the eqs. for v_x and v_y is

$$v_{x,y} = v_\perp \exp(\pm i \omega_c t + i \delta_{x,y})$$

$\pm$ denotes the sign of q

We may choose the phase δ so that

$$v_x = v_\perp e^{i\omega_c t} = \dot{x} \qquad (Eq.\ 1)$$

where $v_\perp$ is a positive constant denoting the speed in the plane perpendicular to $\vec{B}$.

Then $$v_y = \frac{m}{qB} \cdot \dot{v}_x = \pm \frac{1}{\omega_c} \dot{v}_x = \pm \frac{1}{\cancel{\omega_c}} i \cancel{\omega_c} v_\perp e^{i\omega_c t} =$$

$$= \pm i v_\perp e^{i\omega_c t} = \dot{y} \rightarrow (Eq.\ 2)$$

Integrating eqs. 1 and 2, we have

$$x - x_0 = -i \frac{v_\perp}{\omega_c} e^{i \omega_c t} \qquad y - y_0 = \pm \frac{v_\perp}{\omega_c} e^{i \omega_c t}$$

Eqs. (3)

We define the Larmor radius to be

$$r_L \equiv \frac{v_\perp}{\omega_c} = \frac{m v_\perp}{|q| \cdot B}$$

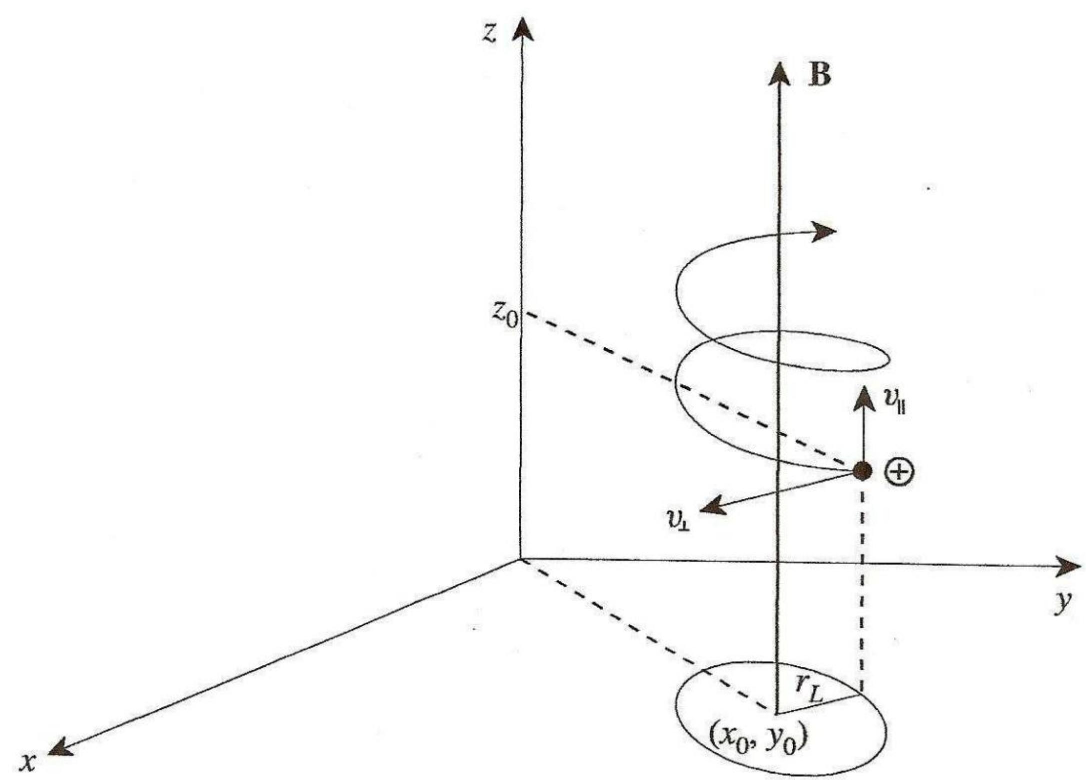
Taking the real part of Eqs. (3) we have

$$x - x_0 = r_L \cdot \sin(\omega_c t) \qquad y - y_0 = \pm r_L \cos(\omega_c t)$$

In the z direction we have $\dot{v}_z = 0 \Rightarrow v_z = \text{constant}$

These equations describe an helical motion along the z direction. In the x-y plane we have a circular orbit around a guiding center (x_0, y_0) with a radius r_L

For positive particles we have a left handed circular motion
For negative particles we have a righ handed circular motion.



1 c –
Relativistic motion in a constant magnetic field.

The equation of motion is

$$\frac{d}{dt}\left(m \gamma \vec{v} \right) = q \left(\vec{v} \times \vec{B} \right)$$

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} = \text{Lorentz factor}$$

$m \equiv$ mass $\qquad q \equiv$ charge

Dotting both sides of the equation with $\vec{v}$ we have

$$\vec{v} \cdot \frac{d}{dt}\left(m \gamma \vec{v} \right) = \underbrace{q\, \vec{v} \cdot \left(\vec{v} \times \vec{B} \right)}_{=0} = 0$$

The left hand side can be written as

$$\vec{v} \cdot \frac{d}{dt}\left(m \gamma \vec{v} \right) = \vec{v}\, m \gamma \dot{\vec{v}} + m \vec{v}^{\,2} \cdot \dot{\gamma} =$$

$$= \frac{1}{2} m \gamma \frac{d(v^2)}{dt} + m \vec{v}^{\,2} \frac{d\gamma}{d(v^2)} \cdot \frac{d(v^2)}{dt} =$$

$$= \underbrace{m \left(\frac{1}{2} \gamma + v^2 \frac{d\gamma}{d(v^2)} \right)}_{\neq 0} \cdot \frac{d(v^2)}{dt} = 0$$

$\hookrightarrow = 0$

Thus we have $v^2 = \text{constant}$ $\Rightarrow$ $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \text{constant}$

Thus, the particle's energy $E = \gamma m c^2$ is a constant

* THE MAGNETIC FIELD CANNOT CHANGE THE KINETIC ENERGY OF THE PARTICLE.
* THIS IS TRUE IN A GENERAL NON-UNIFORM MAGNETIC FIELD, PROVIDED THAT THE ELECTRIC FIELD IS ZERO

Since γm is a constant, we can define $m_{rel} = \gamma m$. With this definition the relativistic equation of motion given before reduces to the non-relativistic equation of motion with $\underline{m}$ replaced by $\underline{m_{rel}}$

Hence, in a uniform field all the results about the non-relativistic motion can be easily generalized to relativistic motion.

In particular we have:

- charged particles move in a helix

- The gyrofrequency is given by $\omega_c \equiv \dfrac{|q| B}{\gamma m}$

- The gyroradius is $r_L \equiv \dfrac{v_\perp}{\omega_c} = \dfrac{\gamma m v_\perp}{|q| B}$

1 d –

Particle motion in a constant electric and magnetic field.

Let us consider $\vec{E}$ = constant and $\vec{B}$ = constant = $B\hat{z}$

- we choose $\vec{E}$ in the x-y plane so that $E_y = 0$
- the z component is unrelated to the transverse components and can be treated separately

The equation of motion is now:

$$m \frac{d\vec{v}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

This gives:

* $\dfrac{dv_z}{dt} = \dfrac{q}{m} E_z \quad \Rightarrow \quad v_z = \dfrac{q E_z}{m} \cdot t + v_{z0}$ acceleration along $\vec{B}$

* $\dfrac{dv_x}{dt} = \dfrac{q}{m} E_x \pm \omega_c v_y$

* $\dfrac{dv_y}{dt} = 0 \mp \omega_c v_x$

The solution of the transverse motion is:

$$v_x = v_\perp e^{i \omega_c t}$$

$$v_y = \pm i v_\perp e^{i \omega_c t} - \frac{E_x}{B}$$

therefore:

- we have an acceleration along $\bar{B}$ due to the z component of $\vec{E}$
- we have a circular Larmor gyration in the x-y plane
- BUT, superimposed to the LARMOR motion there is a DRIFT of the guiding center (in the −y direction for $E_x > 0$)

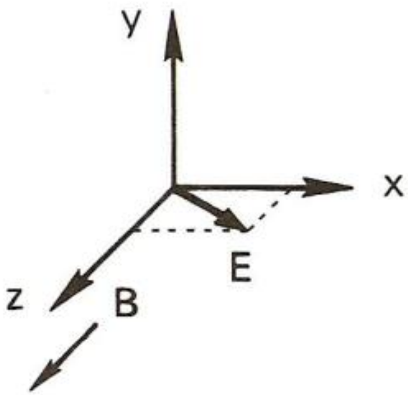
For $\vec{E} = E_x \hat{x}$ and $\vec{B} = B \hat{z}$ we obtained that the drift velocity of the guiding center is

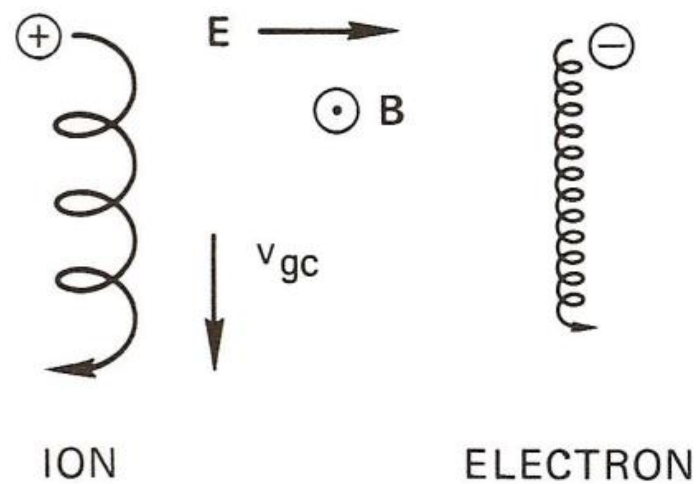
$$v_{gc} = -\frac{E_x}{B}$$

In general, it is easy to show that the DRIFT VELOCITY is:

$$v_E = \frac{\bar{E} \times \bar{B}}{B^2}$$

Notice that v_E is independent of m, q, and $v_\perp$.





DRIFT FOR AN "ION":

- In the first half cycle of the ion's orbit it gains energy from the electric field. $\Rightarrow$ $v_\perp$ increases , r_L increases
- In the second half cycle it losses energy $\Rightarrow$ r_L decreases
- This difference in r_L on the left and righ sides of the orbit causes the drift v_E

DRIFT FOR AN "ELECTRON"

- a negative particle gyrates in the opposite direction but also gains energy in the opposite direction. $\Rightarrow$ it drifts in the same direction as a positive particle.

1 e –

Drift produced by non-electromagnetic forces.

The foregoing results can be easgly generalized to other forces by replacing $q\vec{E}$ in the equation of motion by a general force $\vec{F}$

The guiding center drift caused by $\vec{F}$ is then

$$\vec{v}_F = \frac{1}{q} \frac{\vec{F} \times \vec{B}}{B^2}$$

In particular, if $\vec{F}$ is the force of gravity $m\vec{g}$, there is a DRIFT

$$\vec{v}_g = \frac{m}{q} \cdot \frac{\vec{g} \times \vec{B}}{B^2}$$

- This is similar to the drift $\vec{v}_E$ in that it is perpendicular to both the force and $\vec{B}$.
- But, the drift $\vec{v}_g$ changes sign with the particle's charge.

1 f –
Motion in non-uniform B

For non-uniform $\vec{B}$-fields the problem becomes to complicated to solve exactly.

In general it is necessary to solve numerically the equation of motion

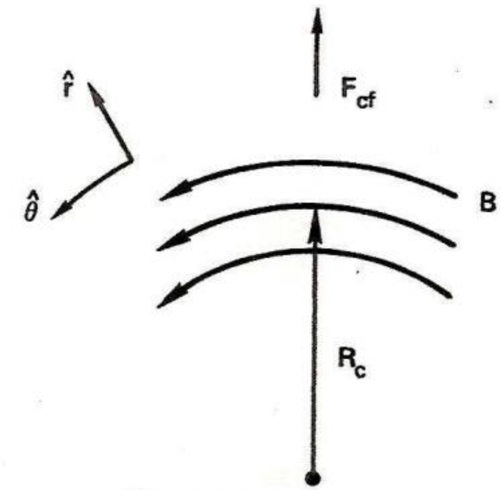
$$m \ddot{\vec{r}} = q \left(\dot{\vec{r}} \times \vec{B}(r) \right)$$

However, for some simple configurations we can obtain interesting conclusions:

- curvature drift
- $\vec{\nabla} |\vec{B}|$ drift
- magnetic mirror.

CURVATURE DRIFT.

We assume the lines of force to be curved with a constant radius of curvature R_c and we take $|B|$ to be constant



A guiding center drift arises from the centrifugal force felt by the particle as it moves along the field lines.

If $v_{\parallel}$ denotes the velocity along B we have

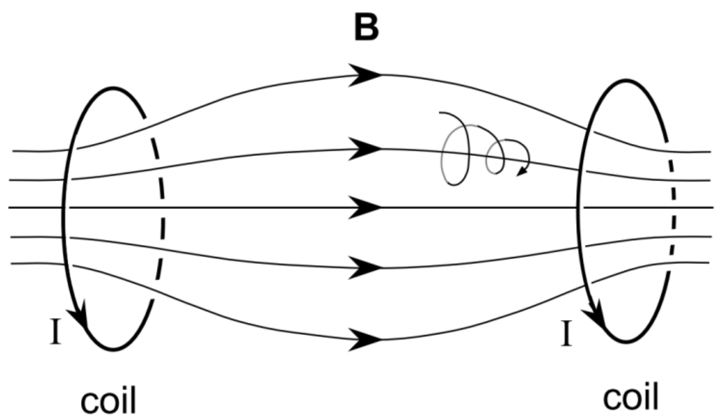
$$\vec{F}_{cf} = \frac{m v_{\parallel}^2}{R_c} \hat{r} = m v_{\parallel}^2 \frac{\vec{R}_c}{R_c^2}$$

Thus we have

$$\vec{v}_R = \frac{1}{q} \frac{\vec{F}_{cf} \times \vec{B}}{B^2} = \frac{m v_{\parallel}^2}{q B^2} \frac{\vec{R}_c \times \vec{B}}{R_c^2}$$

MAGNETIC MIRROR

Let us consider an axisymmetric magnetic field pointed primarily in the z direction and whose magnitude varies in the z direction.



It is possible to show that for most spatially and temporally varying B-fields the quantity

$$\mu \equiv \frac{\frac{1}{2} m v_\perp^2}{B} \equiv \text{magnetic moment}$$

remains invariant as the particle moves (adiabatic invariant)

i.e. $$\frac{d\mu}{dt} = 0$$

- When a particle moves into a region with larger B

  $\Rightarrow$ $v_\perp$ must increase in order to keep $\mu = \dfrac{\frac{1}{2} m v_\perp}{B}$ = constant.

- But, the kinetic energy $E = \frac{1}{2} m v_\parallel^2 + \frac{1}{2} m v_\perp^2$ is a CONSTANT.

  $\Rightarrow$ if $v_\perp$ increases, then $v_\parallel$ must decrease.

- If B grows enough $\Rightarrow$ $v_\parallel$ goes to zero as the particle moves.
  and the particle is <u>reflected</u> back to the region of smaller B

  $\hookrightarrow$ MAGNETIC MIRROR.

Isotropy – Anisotropy of the CR sky

For CRs with high enough energy the gyro-radius is larger than the galactic radius. Those CRs may leave the galaxy:

$$r_g = \frac{p}{qB} = \frac{\gamma m_0 v}{Ze B}$$

Charged particles, on the other hand, are subject to the influence of homogeneous or irregular magnetic fields. This causes the accelerated particles to travel along chaotic trajectories thereby losing all directional information before finally reaching Earth.

This explain why the sky for charged particles with energies < 10^{14} eV appears completely isotropic.

CRs with high enough energy are not as much deflected and thus a certain directionality could be found for them.